Graph Stream Algorithms: *A Survey*

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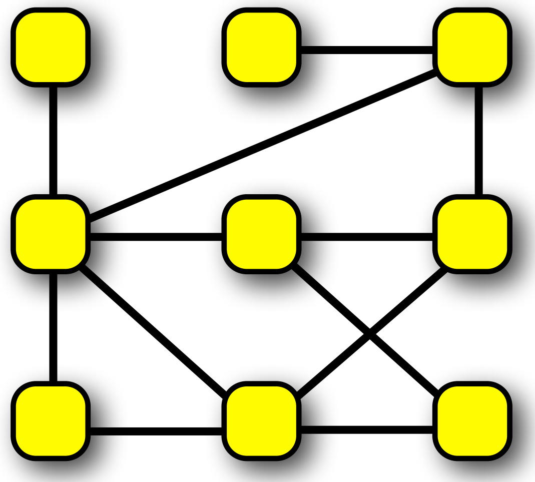
Graph Stream Computational Model

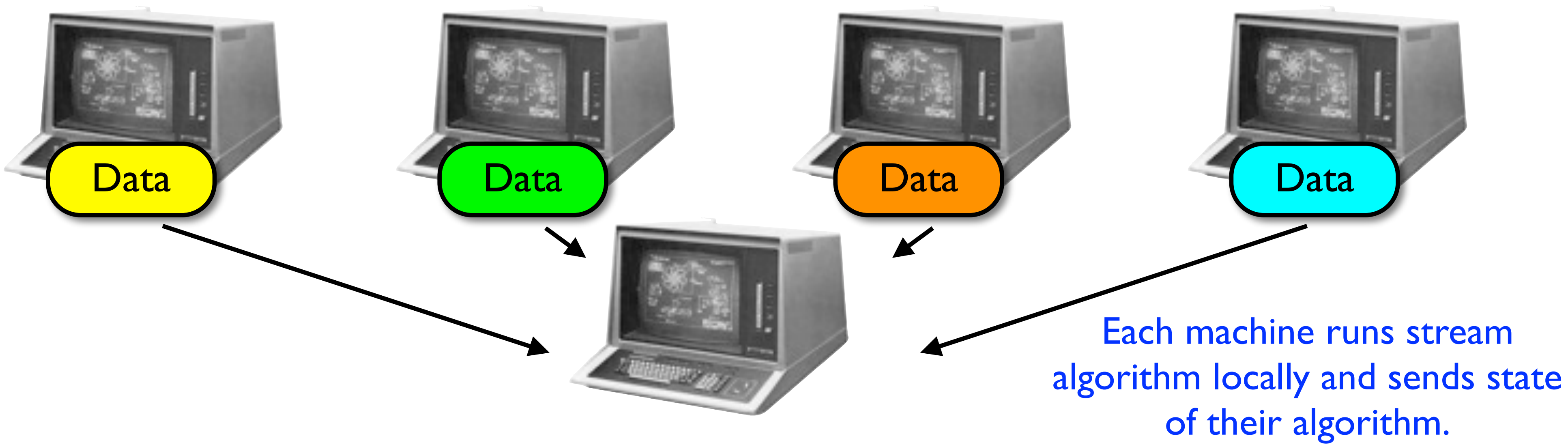
- *Input:* Sequence of edges (e_1, e_2 ...) defines n-node graph G.



- *Goal:* Compute properties of G without storing entire graph.
- *Computational constraints:*
  - i) Limited working memory, e.g., O(n) rather than O(m)
  - ii) Access data sequentially
  - iii) Process each element quickly

Motivation

- *Traditional stream applications:* Network monitoring, reading large data sets from disk, aggregation of sensor readings...
- *Interesting theoretical questions:* How can we summarize graphs? Is there a notion of dimensionality reduction? What types of sampling is possible? Connections to compressed sensing, communication complexity, approximation, embeddings, ...
- *Techniques have wider applications:* E.g., distributed settings,


Data
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Each machine runs stream algorithm locally and sends state of their algorithm.

Outline

- *This Talk:*
  - Algorithms: Summarizing and computing on graph streams
  - Extensions: Sliding windows, extra passes, annotations etc.
  - Future Directions: Directed edges, ordering, stochastic graphs
- *Accompanying Survey:*
  - Includes all references and further details.
  - Feedback welcome...

Graph Stream Algorithms: A Survey

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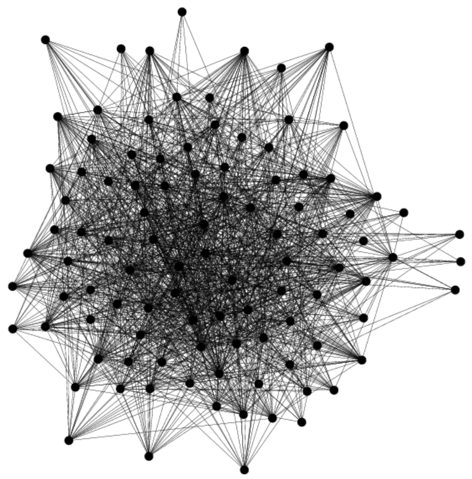
http://people.cs.umass.edu/~mcgregor/papers/13-graphsurvey.pdf

1. Algorithms
2. Extensions
3. Directions

Sparsifiers & Cuts


Original Graph G

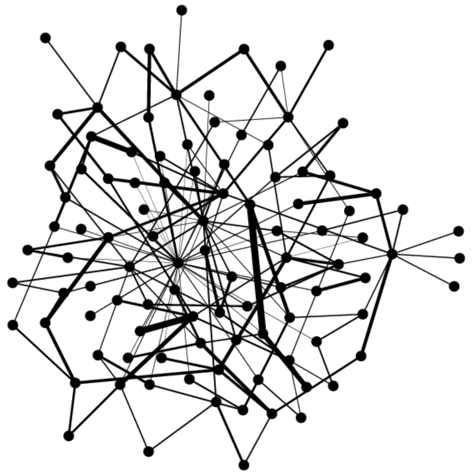
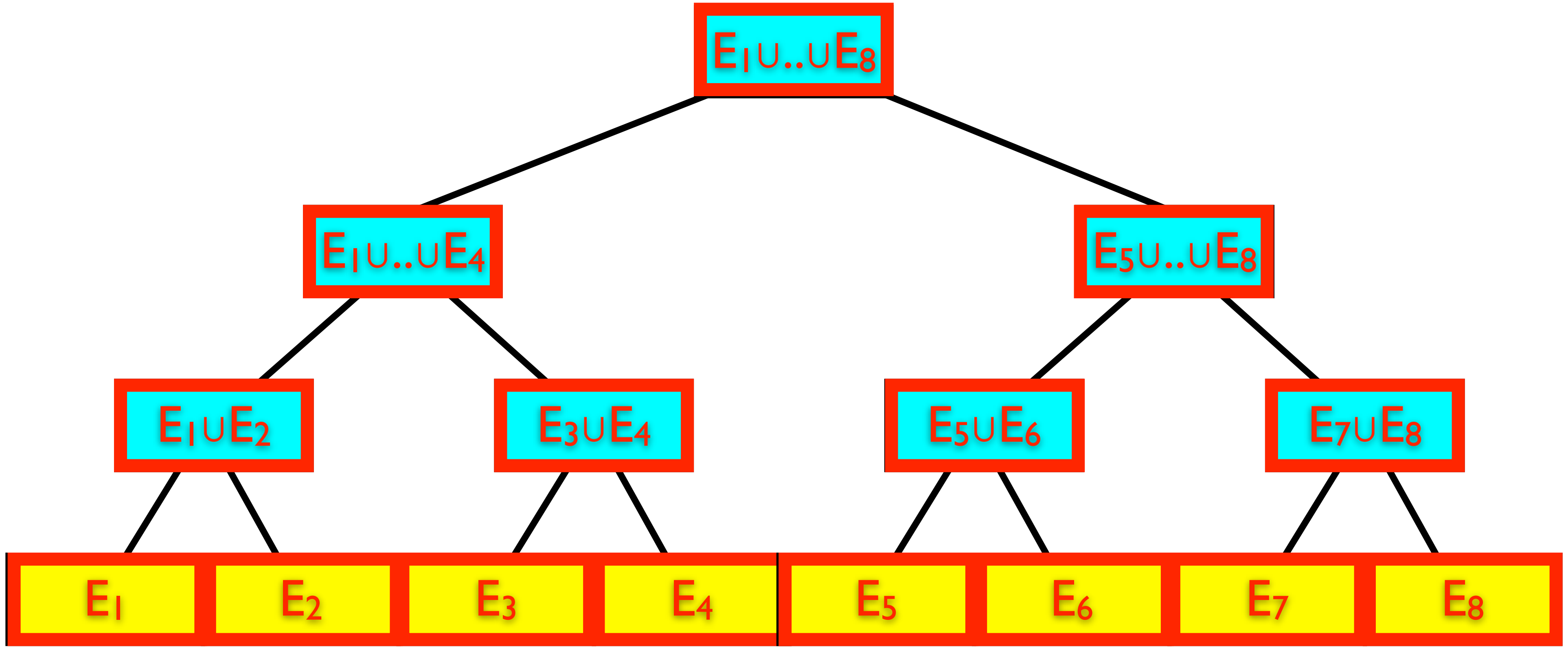

Sparsifier Graph H

Diagram courtesy of Nick Harvey

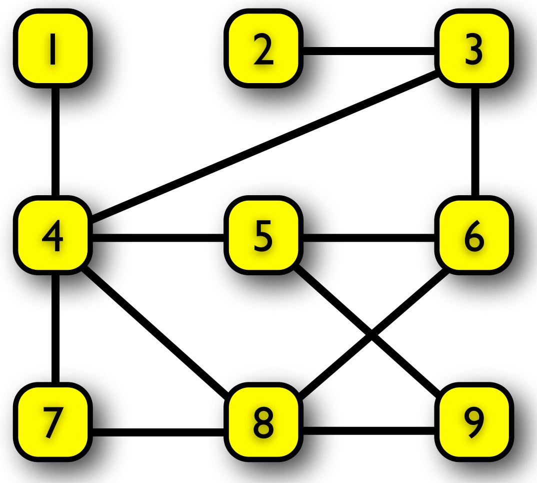
- *Sparsifiers:* A subgraph H is a (1+ε) sparsifier for G if the total weight of any cut is preserved up to a factor 1+ε.
- *Thm:* For any graph G there exists a (1+ε) sparsifier with only $O(\varepsilon^{-2} n)$ edges. Can be constructed efficiently.
- *Thm:* Can construct a (1+ε)-sparsifier of a graph stream using $O(\varepsilon^{-2} n \text{ polylog } n)$ bits of space.

Sparsifier Algorithm

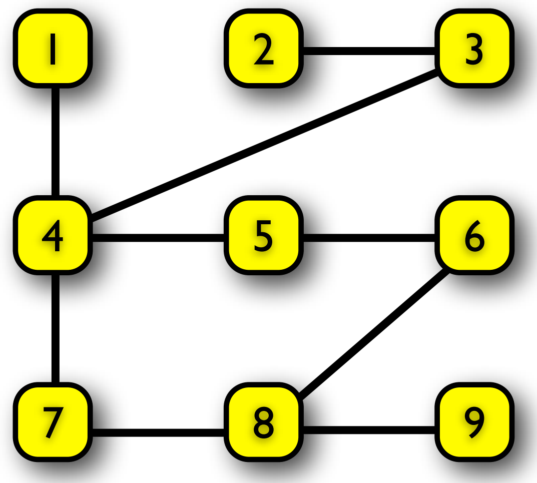


- *Algorithm:* Recursively re-sparsify using any "offline" algorithm.
- *Analysis:* Let d=O(log n) be depth of the tree. Error of a final cut estimate is $(1+\varepsilon)^d$ and we only store d sparsifiers simultaneously.
- Results extend to constructing spectral sparsifiers.

Spanners & Distances



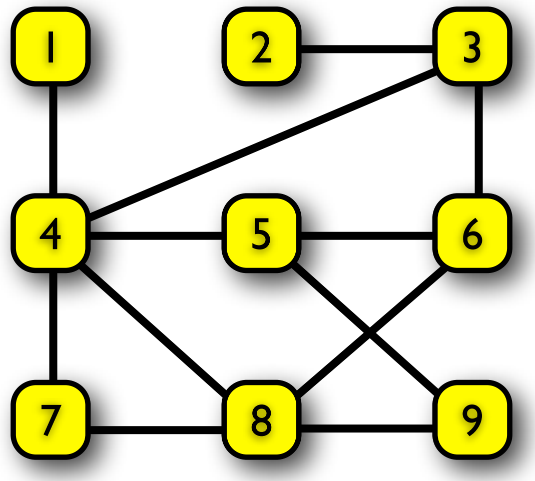
Original Graph G



Spanner Graph H

- *Spanner:* A subgraph H is a k-spanner for G if all graph distances are preserved up to a factor k.
- *Thm:* There is a $O(n^{1+1/t})$ space stream algorithm that constructs a (2t-1)-spanner.

Spanners Algorithm



- *Algorithm:* Store next edge (u,v) unless it completes a cycle of length 2t or less.
- *Lemma:* All distances preserved up to a factor 2t-1 because an edge (u,v) was only ignored if there was already a path of length at most 2t-1 between u and v.
- *Lemma:* At most $(n^{1+1/t})$ edges stored since shortest cycle among stored edges has length at least 2t+1.

Other Algorithms

- *Matchings:*
  - Goal: Find large set of disjoint edges.
  - Results: $\tilde{O}(n)$-space algorithms 2-approx. (unweighted) and 4.91-approx. (weighted). Can do better if edges are grouped together by end-point or arrive in random order.
  - Extensions: $O(1)$ approx. for various sub-modular problems.
- *Counting Triangles:* Estimate the number of triangles (or small cycle or clique etc.). See Seshadhri's talk coming up next...
- *Random Walks:* Simulate length t random walks in $\sqrt{t}$ passes.
- *Other:* Minimum spanning trees, bipartiteness, finding dense components, correlation clustering, independent sets, etc.

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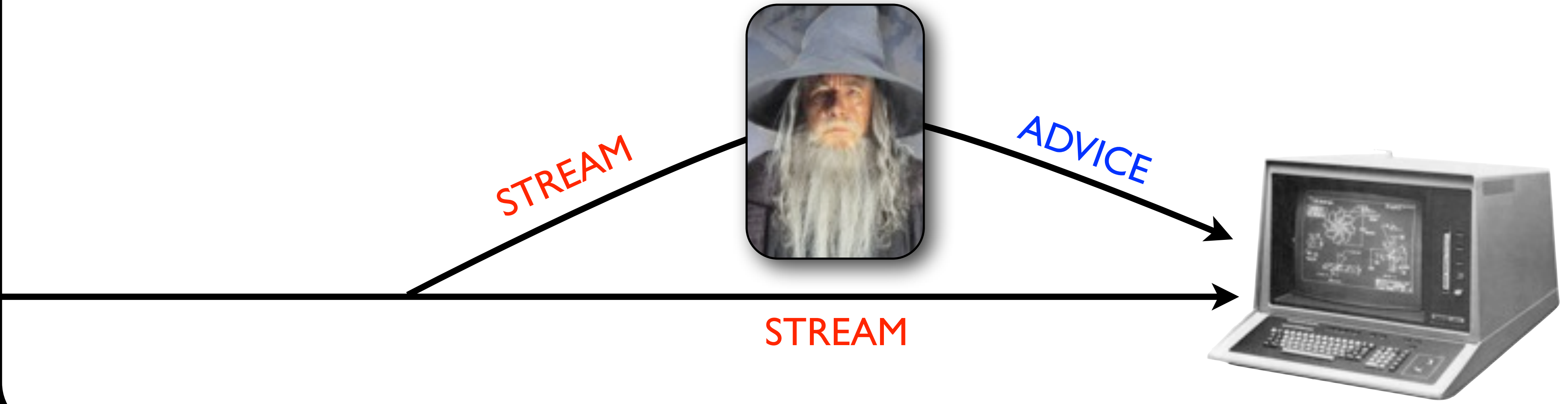
Extensions of Model

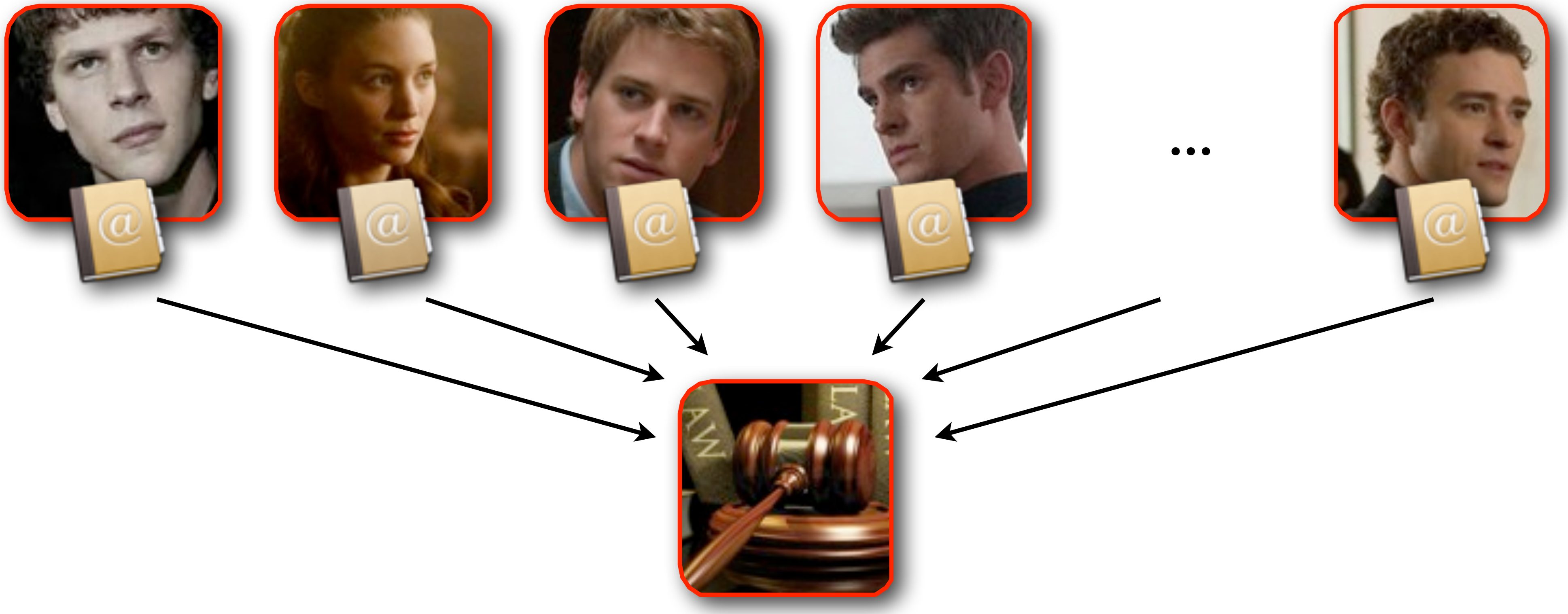
- *Sliding Window:* Infinite stream but only consider graph defined by recent w edges. Can solve most aforementioned problems.
- *Multiple Passes:* What's possible with a small number of stream passes? E.g., can find $1+\varepsilon$ approx. matching in $O(\varepsilon^{-1})$ passes.
- *Annotated Streams:* Suppose a third party "annotates" the stream to assist with the computation. Can we reduce required memory while still verifying correctness.



Dynamic Graphs

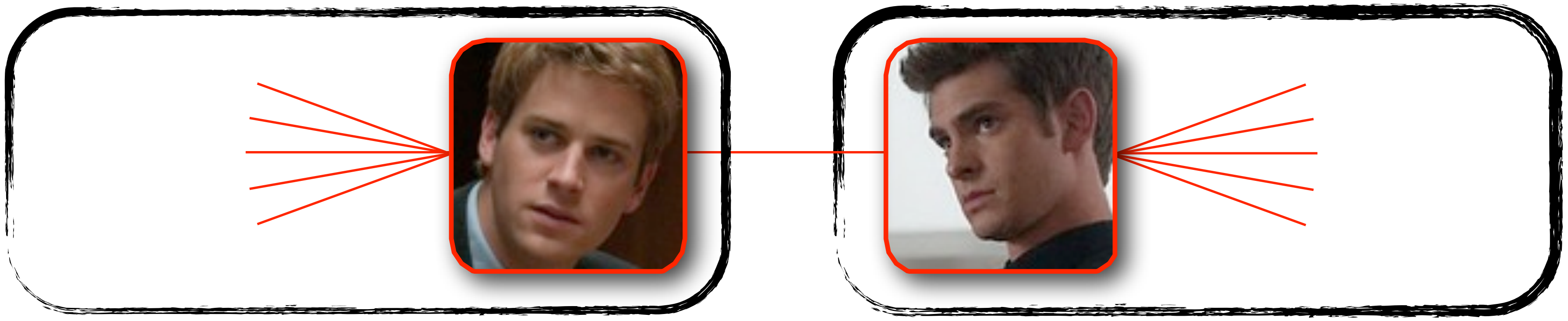
- *Dynamic Graph Streams:* Suppose the stream consists of edges both being added and removed from the underlying graph.
- *Can we maintain a uniform edge sample in small space?*
    - Challenge: The sampled edge we have remembered so far may be deleted at the next step.
    - Result: Can maintain uniform sample in O(polylog n) space via a technique called "l_0 sampling".
- *More powerful sampling techniques:*
    - In O(n polylog n) space, can construct a data structure that returns a random edge across any queried cut.
    - In O(n polylog n) space, can sample edges where (u,v) is sampled w/p inversely proportional to size of min u-v cut.

Distributed Graph Data



- *Setting:* The rows of an adjacency matrix are partitioned between different machines. Equivalently, consider n players each of whom has an "address book" listing their friends.
- *Goal:* Each player sends a "short" message to a third party who then determines if underlying graph is connected.

Distributed Graph Data



- *Appears that some messages need to be $\Omega(n)$ bits:* If there's a bridge (u,v) in the graph, one of the friends needs to mention this friendship but neither friend knows it's a bridge.
- *Thm:* O(polylog *n*) bit messages suffice!
  - Protocol is based on dynamic graph sampling results.
  - Also allows third-party to estimate all cut sizes!

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Open Problems

? *Many specific open questions:*

- Can we construct a spectral sparsifier in $\tilde{O}(n)$-space with deletions? Best algorithm so far uses $\tilde{O}(n^{5/3})$-space.
- Can we construct spanners of sliding window graphs?
- Improve approx. factors for matchings and triangles...

? *Open Problems Wiki:* Large set of open problems in data streams and property testing can be found at:

http://sublinear.info

Future Directions

- *Directed Graphs:* Almost all research to date has considered undirected graphs but many natural graphs are directed. May need multiple passes but O(log n) passes might be sufficient.
- *Stream Ordering:* Consider problems under different orderings, e.g., grouped-by-endpoint, increasing weight, random order.
- *More or Less Space:* Most work has focus on $\tilde{O}(n)$-space algorithms. Can we reduce space-complexity for specific families of graphs? What's possible with slightly more space?
- Explore deeper connections with distributed algorithms, communication complexity, dynamic graphs data structures...

Summary of the Survey

- *Algorithms:* Spanners and sparsifiers capture different properties of the graph. Efficient constructions in streaming model. Other positive results for matchings, triangles, etc.
- *Extensions:* Many variants of the basic model including sliding windows, multi-pass, edge deletions, annotations...
- *Directions:* Improve existing results. Future directions include directed graphs, stream ordering, specific graph families etc.

Thanks!

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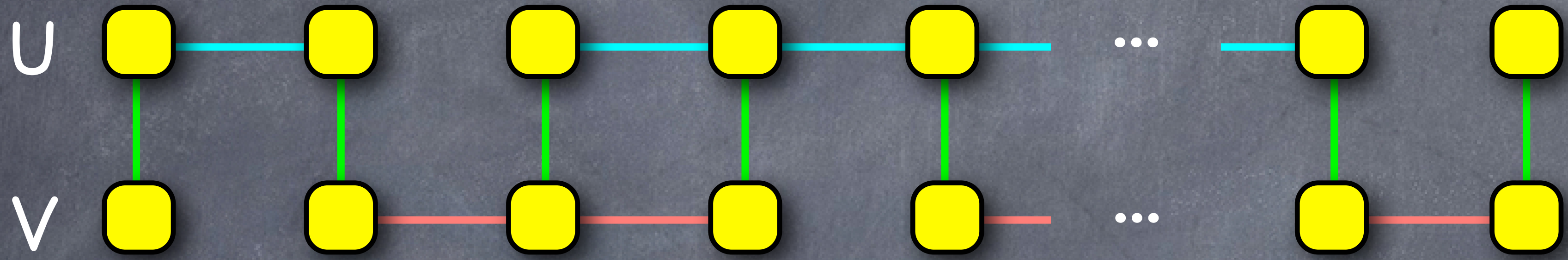
Lower Bound for Connectivity



- Alice and Bob have $x,y \in \{0,1\}^n$. For Bob to check if $x_i=y_i=1$ for some i needs $\Omega(n)$ communication.
- Let A be an s space algorithm for connectivity.
- Consider 2-layer graph (U,V) with $|U|=|V|=n$
- Alice runs A on $E_1=\{u_iv_i: 1\leq i\leq n\}$ and $E_2=\{u_iu_{i+1}:x_i=0\}$
- Send memory to Bob who runs A on $E_3=\{v_iv_{i+1}:y_i=0\}$
- Output of A resolves matrix question so $s=\Omega(n)$.